Triangle Area Graduate Mathematics Conference

North Carolina State University
Saturday, April 2, 2022

The North Carolina State University student chapter of the American Mathematical Society is pleased to host the Spring 2022 Triangle Area Graduate Mathematics Conference, or TAGMaC.

We are pleased to have **Dr. David E. V. Rose** from University of North Carolina, Chapel Hill as our plenary speaker.

The conference will take place in SAS Hall (2311 Stinson Dr, Raleigh, NC 27607) on NC State's main campus. The parking lot next to SAS Hall on Boney Dr is open and free on weekends.

About TAGMaC:

TAGMaC is a bi-annual conference funded by the AMS and hosted on a rotational basis at UNC Chapel Hill, Duke and NC State. It is organized by the AMS student chapters, and its principal objective to expose graduate students of the regional schools to a broad variety of research topics in mathematics and to promote collaboration among them.

Because this conference is somewhat uniquely accessible with respect to both geography and accessibility, we are pleased to open up TAGMaC to advanced undergraduate students from in and around the triangle as attendees.

Schedule

** - accessible to undergraduates

Time	Location	Presenter	Title
9:00-9:30	1st floor		**Registration & Breakfast**
9:30-9:35	1102		**Opening Remarks**
9:35-10:00			**Contributed Talks — Parallel Session 1**
	1108	Erica Swain	Geometric Crystals and their Ultradiscretization
	1218	Katherine Slyman	Rate and Noise Tipping Working in Concert**
	1220	Benjamin Bechtold	An Introduction to Vector Field Methods for Wave Equations**
10:05-10:30			**Contributed Talks — Parallel Session 2**
	1108	Erik Mainellis	Factor Systems and the Second Cohomology Group of Liebniz Algebras
	1218	Alex Mendez	Climate tipping points and transient CO_2 growth**
	1220	Russell Arnold	Shocks and conservation laws in 1D**
10:35-11:00			**Contributed Talks — Parallel Session 3**
	1108	Maria Davis	Dual Nonnegativity Certificates in Polynomial Optimization
	1218	William Reese	Hyper Differential Sensitivity Analysis for Inversion in Land-Ice Dynamics**
11:15-12:15	1102		**Plenary Talk**
		David E. V. Rose	Quantum knot invariants and webs
12:15-1:05	1st floor		**Lunch**
1:05-1:30			**Contributed Talks — Parallel Session 4**
	1108	Joe Johnson	Reverse Plane Partitions of Rectangle and Trapezoid Posets
	1218	Cole Butler	Gene drives and the consequences of over-suppression**
	1220	Geneva Hall	Arnold's Cat Map**
1:35-2:00			**Contributed Talks — Parallel Session 5**
	1108	David White	Symplectic Instanton Knot Homology**
	1218	Ananta Acharya	The diffusive Lotka-Volterra competition model in fragmented patches I: Coexistence**
	1220	Rena Chu	Counterexamples for high-degree generalizations of the Schrödinger maximal operator**

After 2:00, attendees are welcome to socialize in the first floor lobby, any of the rooms where presentations were given, or the 4th floor lounge (SAS 4104).

Plenary Talk

David E. V. Rose — University of North Carolina, Chapel Hill

Title: Quantum knot invariants and webs

Abstract: We'll discuss invariants of knots and links that arose from the field of "quantum topology" in the 1980's (and also briefly comment on their modern analogues: link homology theories). We'll focus on the simplest and most-famous example: the Jones polynomial, and discuss how a theorem relating topology and representation theory (of traceless 2x2 matrices) underlies an elementary description of this invariant. We'll finish by discussing a recent result of the speaker that establishes an analogue of this theorem for symplectic matrices, answering a 25 year old question of Kuperberg.

Contributed Talks

** - accessible to undergraduates

Parallel Session 1 — 9:35-10:00am

Geometric Crystals and their Ultradiscretization 1108

Erica Swain — North Carolina State University

Abstract: Geometric crystals are a geometric analogue of Kashiwara crystals. The ultradiscretization functor maps between the categories of positive geometric crystals and Kashiwara crystals. A conjecture by Kashiwara, Nakashima and Okado posits a surprising connection between the geometric crystal and special types of Kashiwara crystals. I will give a basic overview of these ideas as well as show some results in the efforts to prove this conjecture.

Rate and Noise Tipping Working in Concert** 1218

Katherine Slyman — University of North Carolina, Chapel Hill

Abstract: Rate-induced tipping occurs when a system stops tracking a continuously changing quasi-steady state once the parameter drift speed reaches a certain critical rate, while noise-induced tipping occurs when there are random transitions between two attractors of the underlying deterministic system. While rate-induced tipping does not require any random fluctuations within the system, there can be an interplay between the noise and ramp parameter, resulting in tipping of the system before the critical rate is reached. Building on the work of Paul Ritchie and Jan Sieber, we consider a one-dimensional differential equation with additive noise and a ramp parameter, where there is deterministic rate-induced tipping between two saddle equilibria. We show there exists a heteroclinic connection between these saddle points, which corresponds to the most probable path between these two points for a ramp parameter less than the critical rate. We use the Wazewski principle and symmetry of the unstable and stable manifolds of our two saddles to show this heteroclinic connection exists, as well as consider numerical simulations for verification and visualization of this most probable path.

An Introduction to Vector Field Methods for Wave Equations** 1220

Benjamin Bechtold — University of North Carolina, Chapel Hill

Abstract: Since the 1980s, the Method of Invariant Vector Fields has played a huge role in the study of nonlinear wave equations. In essence, it takes advantage of the symmetries of Minkowski space to derive powerful inequalities which can be used to prove a number of major results, including global existence for solutions to large classes of nonlinear wave equations in four or more space dimensions. In this talk, I will introduce the method and the geometric and analytical ideas behind it, as well as obstacles to its use. Time permitting, I will also discuss modern modifications of this technique that can overcome some of those obstacles and allow us to study three dimensional wave equations and equations on curved spacetimes.

Parallel Session 2 — 10:05-10:30am

Factor Systems and the Second Cohomology Group of Leibniz Algebras 1108

Erik Mainellis — North Carolina State University

Abstract: Factor systems are a tool for working on the extension problem for algebraic structures such as groups, Lie algebras, and Leibniz algebras. Fixing a pair of Leibniz algebras A and B, there is a correspondence between factor systems and extensions of A by B. This correspondence is strengthened by the fact that equivalence classes of factor systems correspond to those of extensions. Under this correspondence, central extensions give rise to 2-cocycles while split extensions give rise to 2-coboundaries. We thus have a notion of the second cohomology group of A with coefficients in B.

Climate tipping points and transient CO2 growth** 1218

Alex Mendez — North Carolina State University

Abstract: We study the mitigation of climate tipping point transitions using an energy balance model. We model the CO2 concentration with a stochastic delay differential equation (SDDE), accounting for various carbon emission and capture scenarios. The resulting coupled system of SDDEs exhibits a tipping point phenomena: if CO2 concentration exceeds a critical threshold, the temperature experiences an abrupt increase of about six degrees Celsius. We show that the CO2 concentration exhibits a transient growth which may cause a climate tipping point, even if the concentration decays asymptotically. We derive a rigorous upper bound for the CO2 concentration, providing sufficient conditions for evading the climate tipping point.

Joint with: Mohammad Farazmand

Shocks and conservation laws in 1D** 1220

Russel Arnold — University of North Carolina, Chapel Hill

Abstract: Wave propagation for dissipative or dispersive systems can be studied via hyperbolic reduction. When the asymptotic assumptions leading to the reduced system cease to be valid, discontinuities emerge in the hyperbolic system. In some cases the resulting jump discontinuities can be restricted in such a way that the behavior of the underlying system is still represented. This state of affairs is known as shock formation. We will consider the relationship of shock formation to conservation laws.

Joint with: Roberto Camassa

Parallel Session 3 — 10:35-11:00am

Dual Nonnegativity Certificates in Polynomial Optimization 1108

Maria Davis — North Carolina State University

Abstract: One way to show that a polynomial is nonnegative over a compact semialgebraic set is to show that the polynomial can be written as a weighted sum of squares (WSOS). Building on the theory of interior-point methods for convex optimization, we introduce the concept of dual certificates, which allows us to interpret vectors from the dual of the sum-of-squares cone as rigorous nonnegativity certificates of a WSOS polynomial. Whereas conventional WSOS certificates are rearrangements of the polynomials they certify, dual certificates are distinct from the certified polynomials; moreover, each dual certificate certifies a full-dimensional convex cone of WSOS polynomials. For a theoretical application of dual certificates, we give a short new proof of Powers' theorems on the existence of rational WSOS certificates of positive polynomials. For a computational application, we show that exact WSOS certificates can be constructed from numerically computed dual certificates at little additional cost, without any rounding or projection steps applied to the numerical certificates. We also present an algorithm for computing the optimal WSOS lower bound of a given polynomial along with a rational dual certificate, with a polynomial-time computational cost per iteration and linear rate of convergence.

Joint with: Dávid Papp

Hyper Differential Sensitivity Analysis for Inversion in Land-Ice Dynamics** 1218

William Reese — North Carolina State University

Abstract: Devoloping high fidelity ice sheet models is important for modeling global climate and predicting sea-level rise. Modeling large ice sheets such as Antartica or Greenland, require extensive computational resources to solve a systems of nonlinear PDEs on fine scale meshes. Additionally, there are uncertainties associated with spatially distributed model parameters such as basal sliding, bedrock topography, and source terms. In this work we utilize Hyper Differential Sensitivity Analysis (HDSA) to provide insight into the sensitvity of estimated bedrock topography with respect to other model parameters. This is accomplished by solving an inverse problem for bedrock topography constrained by the shallow ice model and then differentiating through the optimality system to determine the influeuence of the other model parameters on the estimated bedrock topography. Our analysis is performed on a 550 x 450 km region of Greenland over a time interval of 10 years.

Joint with: Joseph Hart, Bart van Bloemen Waanders, John Jakemen, Mauro Perergo, Arvind Saibaba

Parallel Session 4 — 1:05-1:30pm

Reverse Plane Partitions of Rectangle and Trapezoid Posets 1108

Joe Johnson — North Carolina State University

Abstract: The rectangle poset and the action of rowmotion on its labelings are two well studied objects in dynamical algebraic combinatorics, but work involving the trapezoid poset has proven more difficult. The rectangle and trapezoid posets are very similar; they have the same number of reverse plane partitions at each height and it is conjectured that the orbit structures of rowmotion on these two posets are the same. We give a bijection between the reverse plane partitions of the rectangle and trapezoid, and some partial results towards proving equivariance with respect to rowmotion.

Joint with: Ricky Liu

Gene drives and the consequences of over-suppression** 1218

Cole Butler — North Carolina State University

Abstract: Suppression gene drives (SGDs) spread a deleterious genetic cargo through a population by biasing their own inheritance. This technology offers a promising solution to the burden posed by crop pests and vectors of important human diseases. Presently, theoretical and experimental studies favor SGD constructs that quickly eradicate a population. If drive killing occurs faster than drive spreading, however, the target species can be locally eradicated. In the presence of migration from a non-controlled region, local eradication risks the re-invasion of wild-type immigrants, consequently undermining or even reversing suppression efforts. How might we balance drive lethality with target population permanence in the presence of bidirectional migration? In this work, we seek to answer this question for select SGDs. We use a patch-based model to account for heterogeneity in population density across a landscape. Bidirectional migration is considered between a target and non-target population. SGD performance is studied as migration levels vary, and we seek to establish under what conditions the drive persists in a suppressed target population while remaining robust to migration.

Arnold's Cat Map** 1220

Geneva Hall — University of North Carolina, Chapel Hill

Abstract: The heart of classical mechanics is to determine the future of a dynamical system given initial conditions. I have studied the properties of a map published by Vladimir Arnold in 1968 now called "Arnold's cat map." This map is a transformation of the two dimensional torus $\mathbb{T}^2$ into itself that can be written $\Gamma(x, y) = (2x + y, x + y) \mod 1$. This map is a hyperbolic toral automorphism, a subset of the class of Anosov diffeomorphisms, that has interesting ergodic, geometric, and topological properties.

Parallel Session 5 — 1:35-2:00pm

Symplectic Instanton Knot Homology 1108

David White — North Carolina State University

Abstract: Motivated in part by the Atiyah-Floer conjecture, there have been a number of constructions of Lagrangian Floer homology invariants for 3-manifolds which are defined in terms of symplectic character varieties arising from Heegaard splittings. We develop a relative variant of one of these homologies, due to H. Horton, for knots in 3-manifolds. The foundational machinery is drawn from knot Floer homology (HFK) and from the extensive literature on the symplectic properties of character varieties with holonomy restrictions.

The diffusive Lotka-Volterra competition model in fragmented patches I: Coexistence** 1218

Ananta Acharya — University of North Carolina, Greensboro

Abstract: We take a competitive model which describes the steady states of two species u and v competing in a habitat Ω. Here b_1, b_2 represent the strengths of competition, λ represents a patch size measure, and γ_1, γ_2 are related to the hostility of the exterior domain. We analyze the positive solutions of the model as the parameters b_1, b_2 and γ_1, γ_2 vary.

Joint with: S. Bandyopadhyay, J. Goddard II, A. Muthunayake, R. Shivaji

Counterexamples for high-degree generalizations of the Schrödinger maximal operator** 1220

Rena Chu — Duke University

Abstract: In 1980 Carleson posed a question on the minimal regularity of an initial data function in a Sobolev space $H^s(\mathbb{R}^n)$ that implies pointwise convergence for the solution of the linear Schrödinger equation. After progress by many authors, this was recently resolved (up to the endpoint) by Bourgain, whose counterexample construction for the Schrödinger maximal operator proved a necessary condition on the regularity, and Du and Zhang, who proved a sufficient condition. Analogues of Carleson's question remain open for many other dispersive PDE's. We develop a flexible new method to approach such problems, and prove that for any integer $k \geq 2$, if a degree k generalization of the Schrödinger maximal operator is bounded from $H^s(\mathbb{R}^n)$ to $L^1(B_n(0,1))$, then $s \geq \frac{1}{4} + \frac{n-1}{4((k-1)n+1)}$. In dimensions $n \geq 2$, for every degree $k \geq 3$, this is the first result that exceeds a long-standing barrier at 1/4. Our methods are number-theoretic, and in particular apply the Weil bound, a consequence of the truth of the Riemann Hypothesis over finite fields.

Joint with: Chen An, Lillian B. Pierce